Using Simulation Models Structure for optimization: an application to queuing systems

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The Discrete Event Optimization (DEO) framework has recently established a ground for the integrated simulation and optimization of discrete event systems (DES's). The basic idea is to exploit the possibility to write mathematical programs that generate DES's sample paths to solve complex optimization problems.
Although DEO has been successfully applied to several queueing problems, the solution of the mathematical models is too computationally expensive. This is due to the fact that the size of the problem to solve grows not only with the control variables but also with the number of events being considered. In this regard, we aim at contributing to the large family of simulation–optimization approaches by generating *cuts* that **efficiently reduce the search space** and:

- Exploit the structure of the simulation model, thus avoiding the exponential growth of the optimization model in the number of events;
- Result general enough to be coupled with several sampling schemes;
- Result flexible enough to be applied to a large class of problems

In order to do so, we investigate the Benders Decomposition (BD) as means to solve the DEO models, **but we generate** the cuts using a **simulation**-based approach.

We show the concept behind the proposed approach through the Server Allocation Problem (SAP) in a $G/G/m$ system. The Server Allocation Problem deals with the choice of the minimum number m of servers needed to have the average waiting time smaller than a given upper bound. The incoming buffer has infinite capacity and the inter-arrival time and process time are stochastic processes following general distributions. We formulate the SAP as a Mathematical Programming (MP) model based on the Discrete Event Optimization (DEO) framework. A DEO model contains two sets of constraints and variables: one modelling the allocation aspect of the problem (optimization component) and the other modelling system dynamics (simulation component). We apply the BD with the simulation-based cut generation to the SAP. A challenge in the application of the approach is that the simulation component results in a mixed integer programming model, which violates the assumption of the standard BD. However, we observe that the integer variables are parameters for the Benders' subproblems once the master problem is solved.