# Sample Average Approximation Method for Stochastic Programming Problems with Probabilistic Criteria

Sergey Ivanov  Andrey Kibzun
Sobolev Institute of Mathematics
Acad. Koptyug avenue, 4, Novosibirsk, Russia, 630090
sergeyivanov89@mail.ru
Moscow Aviation Institute,
Volokolamskoe shosse, 4, Moscow, Russia, A-80, GSP-3, 125993
kibzun@mail.ru

Stochastic programming problems with probabilistic and quantile criteria [1] are considered. Let a loss function $\Phi(u,x)$ be given, where $u \in U \subset \mathbb{R}^m$ is an optimization strategy, $U$ is a compact set, $x$ is a realization of a random vector $X$. The probability function is defined as $P_\varphi(u) \triangleq \mathbf{P}\{\Phi(u,X) \le \varphi, Q(u,X) \le 0\}$, where $Q(u,x)$ is a given function describing additional probabilistic constraints. We suppose that the functions $Q(u,x)$ and $\Phi(u,x)$ are measurable and lower-semicontinuous in $u \in U$. The quantile function is the minimal level of losses $\Phi(u,x)$ that cannot be exceeded with a fixed probability $\alpha$, i.e. $\varphi_\alpha(u) \triangleq \min\{\varphi \mid P_\varphi(u) \ge \alpha\}$. We consider the probability maximization problem $\max_{u\in U} P_\varphi(u)$ and the quantile minimization problems $\min_{u\in U} \varphi_\alpha(u)$.

According to the Sample Average Approximation Method, the considered problems are approximated by similar problems in which the probability function $P_\varphi(u)$ is replaced by its sample estimator $P_\varphi^{(n)}(u)$, where $n$ is the sample size. Using the result [2], we show that the sequence $\{P_\varphi^{(n)}(\cdot)\}$ hypo-converges to $P_\varphi(\cdot)$ almost surely as $n \to \infty$. The hypo-convergence ensures that all limit points of the sequence of optimal solutions $\{u_\varphi^{(n)}\}$ to the problems $\max_{u\in U} P_\varphi^{(n)}(u)$ are optimal solutions to the probability maximization problem.

The conditions of convergence of optimal solutions $\{u_\alpha^{(n)}\}$ to the minimization problems $\min_{u\in U} \min\{\varphi \mid P_\varphi^{(n)}(u) \ge \alpha\}$ can be obtained from [3] for the case of continuous in $u \in U$ functions $Q(u,x)$ and $\Phi(u,x)$. In this work, we suggest conditions ensuring that all limit points of the sequence of optimal solutions $\{u_\alpha^{(n)}\}$ are optimal solutions to the quantile minimization problem. In these conditions, the function $Q(u,x)$ and $\Phi(u,x)$ are lower-semicontinuous in $u \in U$.

We apply the obtained results to two-stage and bilevel stochastic programming problems with probabilistic criteria.

## References

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