MILP-based models for non-stationary stochastic lot-sizing strategies

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In the literature, there are three commonly adopted policies for solving stochastic lot sizing problems, namely, (s,S) policy, (R,S) policy, and (R,Q) policy. Each of these policies has different advantages and disadvantages. The (s,S) policy is known to be cost-optimal; the (R,S) policy has advantages in organising joint replenishments and shipment consolidation; the (R,Q) policy is appealing in material requirement planning systems, for which order synchronisation is a key concern.

A few efficient inventory models have been emerging in the literature to compute optimal policy parameters with independent and identically distributed demands. However, a number of studies revealed that, in many inventory systems, demands are correlated. In this direction, studies either proved the optimality of the (s,S) policy with Markov-modulated demands or measured the performance of the inventory system in response to the demand correlation. Unfortunately, no efficient approaches have been provided to compute optimal policy parameters, especially, for time-series-based demand process. This motivates our work.

In this paper, we consider a single-item single-stocking location inventory lot-sizing problem. We mainly focus on the (R,S) policy with the first order autoregressive (AR(1)) demand process. We present a mixed integer linear programming (MILP) model to approximate the optimal policy parameters. Our model is built upon the conditional distribution; it employs the piecewise linear approximation presented; it can be easily implemented and solved by using off-the-shelf optimisation software.

In order to explore the computational performance of our model, we design a test bed featuring 405 instances defined over a 10-period planning horizon. On this test bed, we compare against the stochastic dynamic programming model in terms of the behaviour of the optimality gap and the computational time. Our numerical examples are conducted by using the IBM ILOG CPLEX Optimization Studio 12.7 and MATLAB R2016a on a 3.2GHz Intel(R) Core(TM) with 8GB of RAM. Computational experiments demonstrate that the optimality gap of our model is around 1% of the optimal policy cost and the computational time is only a few seconds.

We contribute to literature on stochastic lot sizing by presenting an MILP model integrated the AR(1) demand process for approximating the optimal (R,S) policy parameters. Our model provides tighter optimality gap and reasonable computational time. In addition, our model can be easily adjusted to computing optimal (s,S) policy and (R,Q) policy parameters as well. Apart from that, our model can be applied for the autoregressive (AR(p)) demand process of order p.