Optimization of time-dependent processing rates in stochastic systems

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A key challenge in production and service systems is to adapt the resource capacity to a time-dependent and uncertain demand. Optimizing the changes of the processing rate based on time-dependent demand is a way to improve the performance for such stochastic systems.

In this paper we analyze the influence of demand changes on the optimal decision to change the processing rates of a stochastic production or service system. The aim of this paper is to gain insights on how the information on demand changes influences the optimal control of the processing rate of a stochastic multi-server system. We model that system as a single-stage *M(t)/M(t)/c*-queue. This could either be a make-to-order production system or a multi-server service system. A non-homogeneous Poisson process with instantaneous arrival rate $\lambda(t)$ creates arrivals to this system. The decision is to choose optimal processing rates within a given range. The processing rate is allowed to change in certain pre-defined periods. The objective function considers holding cost proportional to the work-in-process and service cost proportional to the current processing rate. Linear and quadratic holding cost functions are investigated. Additionally, a reward for finished items or served costumers is modeled. The reward per unit is assumed to be constant or to be dependent on the current processing rate.

We present a general integrated decision model for optimizing the processing rates. First, closed form solutions are derived with a single decision on the processing rate under stationary conditions. Second, we analyze the system with time-dependent demand and multiple changes of the processing rates. The time-dependent behaviour is modeled with a fluid approach and a stationary backlog-carryover (SBC) approach. In the deterministic fluid approach, optimal processing rates are derived analytically for constant rewards. The SBC-approach considers both the time-dependent changes and the stochastic nature of the decision problem. Since the processing rates are the decision variables, we develop a new SBC-approach that calibrates the period length dependent on the result of the optimization. This allows for general reward functions that depends on the processing rates.

For the stationary system it is found that the optimal solution in a stochastic environment add a “safety capacity” to the optimal stationary solution in the deterministic model. The decision on the processing rate in a current period depends on future demand changes in stochastic systems with both constant and rate-dependent reward. We show numerically that an increasing arrival rate leads to a change in the processing rate some periods before the change in the arrival rate. Therefore, a forecasted change in the demand is anticipated periods before the change happen. We intensively discuss the impact of such future demand changes on the optimized processing rates of previous periods. Additionally, a numerical study shows the reliability of the new approach.