A Two-stage stochastic optimization framework for train rescheduling considering backup trains in a metro line with disruptions

Jiateng Yin, Yihui Wang*, Lingyun Meng, Shuai Su, Jing Xun, Tao Tang
State Key Laboratory of Rail Traffic Control & Safety,
Beijing Jiaotong University, Beijing 100044
{jiatyin, yihui.wang, lymeng, shuaisu, jxun, ttang}@bjtu.edu.cn

In heavily congested metro lines of large cities, unexpected disruptions often occur, which may cause severe delays to trains, blockages on tracks and stations, and reductions of passenger service quality. When disruptions happen, the dispatchers reschedule timetables and try to recover to the normal operation state as soon as possible in order to alleviate the crowdedness of passengers at the platforms and onboard trains, especially during the peak hours. Since most of metro lines have several storage tracks in the middle of the lines, the dispatchers can dispatch the backup trains at those storage tracks to fill the gap between oversaturated passenger demands and shortage of transport capacity in disrupted situations. Nevertheless, due to the uncertain characteristics of disruptions in a metro line, involving the occurring position and duration time of disruptions, it is a challenging issue to simultaneously determine (1) the train storage tracks to hold backup train(s) under normal operation circumstances and (2) the rescheduled train timetable in case of a disruption, i.e., the arrival and departure times of both in-service trains and backup trains from the storage tracks.

To capture the uncertain characteristics of disruptions (i.e., the position and duration of disruptions) in a metro line, this study employs a sample-based method to describe the randomness of disruptions, in which each sample corresponds to the occurring position and duration time of disruptions consideration the correlation of randomness. This kind of representation typically can include the real-world historical data of Beijing metro lines. Then, we aim to develop a two-stage stochastic programming model with the minimized expected train delay time and cancelled service trains compared with the original timetable. Specifically, the first stage of the model determines the optimal numbers of backup trains needed and their corresponding parking tracks to balance the backup train utilization cost and expect train delay penalty (i.e., train delay time and cancelled service trains). Meanwhile, the second stage provides the optimal train rescheduled timetable, i.e., the orders and departure times of in-service trains and added backup trains in the first stage for evaluating the given number and parking positions of these backup trains.

To solve the formulated model, we first propose a number of novel reformulation techniques to establish equivalent mixed integer linear programming models that can be potentially handled by some exact optimization methods (e.g., branch and bound) or commercial solvers (e.g., CPLEX and Gurobi). Then, a Lagrangian decomposition approach is further developed to dualize the hard constraints that couple different samples. In this way, the relaxed model is decomposed into a series of sub-problems that are much easier to be solved to optimality. We also develop several heuristic algorithms to adjust the solution of relaxed models into a feasible solution of the primal problem. To further enhance the computational efficiency for the Lagrangian multiplier updating process, parallel computing and OpenMP technique on a Visual Studio 2012 Platform coded by C++ are implemented for generating the lower bound at each iteration in the Lagrangian relaxation algorithm. Finally, numerical experiments based on the operation data of Beijing Metro Line 6 are implemented to verify the effectiveness of the proposed two-stage stochastic model and solution approaches.

References

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* Corresponding author. This research was supported by the National Natural Science Foundation of China (Nos. 61503020 and 71571012) and the Beijing Municipal Commission of Education (Contract No. I15H100010).